Graphical Language for Clifford Hermiticians in Quantum Computing

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ZX-Calculus

The ZX-Calculus[1] is a graphical language initially used to describe pure quantum operators. Its main feature – compared with quantum circuits – is that it comes equipped with an intuitive equational theory i.e. a set of graphical transformations that turns any diagram into an equivalent one (i.e. which represents the same quantum operator). We say that an equational theory is *complete* when all the possible tranformations are captured by the equations of the equational theory. Complete equational theories were hence found for several restrictions of the language[2].

Adding a generator  to the language makes it possible to amend for mixed states and completely positive maps, in other words, this allows us to represent measurements inside the graphical language. We may then extend the equational theories to capture the interaction between this generator and the previous ones. Again, complete equational theories were found[3].

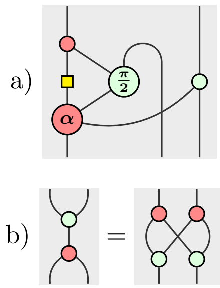

Fig. 1. a) example of a ZX-diagram, and b) an equation in the equational theory

Extension to Hermitian Operators

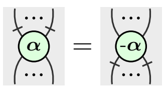

Fig. 2. a new equation

Recently, the possiblity to extend the language to *anti-unitaries* was studied[4], and lead to the addition of a new generator  used to represent the conjugation operator. Interestingly, this generator subsumes  – the latter can be decomposed with  and other generators from the initial language. It was shown that the diagrams we get with the new generator represent exactly Hermitian operators and Hermiticity-preserving superoperators. A complete equational theory was found for several restrictions of the language, but, interestingly, not for the most natural one to look at first: the Clifford fragment, obtained by restricting the angles to multiples of $\frac{\pi}{2}$.

Internship Goal

We propose in this internship to look for a complete equational theory for Clifford Hermitian operators and Hermiticity-preserving superoperators. From the previous result, we can already infer some useful equations for our problem. It remains to see if they are enough or if we need to add new ones to get completeness. To that aim, we will look for normal forms for Clifford diagrams with  and try to extend them to the language at hand. Then we will have to show that every Clifford diagram with the new generator can be turned in normal form.

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[1] `https://en.wikipedia.org/wiki/ZX-calculus`, `https://arxiv.org/abs/2012.13966`
[2] `https://arxiv.org/abs/1307.7025`, `https://arxiv.org/abs/1705.11151`, `https://arxiv.org/abs/1812.09114`
[3] `https://arxiv.org/abs/1902.07143`
[4] Work in progress, not published yet